EXPERIMENTAL-THEORETICAL INVESTIGATION OF FLOW DISTRIBUTION IN A POROUS CHANNEL

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By considering the boundary-value problem of the motion of a viscous fluid in a channel with porous walls, an approximate description of the distribution of axial velocity and static pressure is obtained.

In solving many important problems it is necessary to know how the pressure and velocity vary along a channel with porous walls and a dead end. Turbulent flow in channels with outflow varying along the length is particularly complex and little studied.

In [1] the equation of motion of a fluid with a variable flow rate along the path [2]

$$\frac{dp}{\rho}+\beta WdW+Wd\beta+\frac{\beta W(W-\Theta)\,dG}{G}+\lambda\frac{W^2}{2}\cdot\frac{dx}{D}=0$$

is used to determine the longitudinal pressure gradient in the presence of outflow (inflow).

The solution of this equation requires data on the variation of β, θ, and λ along the porous channel.

Experimental studies of the structure of turbulent flow in a circular pipe established the nature of the variation of the momentum flux factor β for both a constant [3] and a variable [4] rate of outflow of gas.

For a nonuniform rate of outflow β at first increases to $\beta = 1.042$ for $x/D = 4$, decreases to $\beta = 1.02$, and then remains constant almost to the dead end where it increases sharply. Thus, β differs only slightly from unity.

It is very difficult to establish the functional dependence of the friction factor λ on other factors.

In recent theoretical papers attempts have been made to use the idea of a mixing length, refined near the wall by means of a damping factor [5], for all types of turbulent boundary layers.

It has been proposed to calculate the friction factor in porous channels with a uniform distribution by an expression in which the friction factor is a linear function of the coefficient of outflow

$$\lambda=\lambda_0+5.54\,\frac{v_w}{\overline{W}}\,.$$

Reliable experimental data are needed to confirm this expression.

If we take $\beta = 1$, $\Theta = cW$ $(0 \le c \le 1)$, and the known value of $G = (\pi D^2/4)W\rho$, the equation of motion of a fluid with a variable flow rate takes the form

$$\frac{dp}{\rho}+(2-c)\,WdW+\lambda\,\frac{W^2}{2}\cdot\frac{dx}{D}=0. \tag{1}$$

On the one hand, the radial velocity at the wall can be determined from the discharge formula

$$v_w=\varepsilon_f\sqrt{2p/\rho\,(1+\zeta)}\,. \tag{2}$$

On the other hand,

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$$v_W = -\frac{D}{4} \cdot \frac{dW}{dx}. \tag{3}$$

Using $\bar{f} = \Sigma f_{hol}/F_k = 4\varepsilon_f L/D$, we have from Eqs. (2) and (3)

$$p = (dW/dx)^2 (1 + \zeta)/2\bar{f}^2. \tag{4}$$

Differentiating (4), substituting into (1), and making some transformations, we obtain

$$\frac{d^2W}{dx^2}\frac{dW}{dx} + \frac{16\varepsilon_f^2 (2 - c)}{1 + \zeta}\left(\frac{L}{D}\right)^2 W \frac{dW}{dx} + \frac{8\varepsilon_f^2 \lambda}{1 + \zeta}\left(\frac{L}{D}\right)^3 W^2 = 0.$$

To determine the friction factor we start from the equation

$$\lambda = \lambda_0 + M \frac{v_W}{\bar{W}}.$$

If we substitute $8c + M = N$ and introduce the dimensionless variables $X = x/L$ and $U = W/W_0$,

$$\frac{d^2U}{dX^2}\frac{dU}{dX} + \frac{16\varepsilon_f^2 (2 - N/8)}{1 + \zeta}\left(\frac{L}{D}\right)^2 U \frac{dU}{dX} + \frac{8\varepsilon_f^2 \lambda_0}{1 + \zeta}\left(\frac{L}{D}\right)^2 U^2 = 0.$$

Finally, we obtain the boundary-value problem

$$\frac{d^2U}{dX^2}\frac{dU}{dX} + aU\frac{dU}{dX} + bU^2 = 0; \quad U(0) = 1; \quad U(1) = 0, \tag{5}$$

where

$$a = \frac{16\varepsilon_f^2}{1 + \zeta}\left(\frac{L}{D}\right)^2 (2 - N/8); \quad b = \frac{8\lambda_0 \varepsilon_f^2}{1 + \zeta}\left(\frac{L}{D}\right)^3.$$

The coefficient N must be determined from the experimental data. It is impossible to find an exact analytical solution suitable for analysis.

We solve this equation by the power series method proposed in [6]. We seek a series solution in the form

$$U = \sum_0^\infty C_n (1 - X)^n. \tag{6}$$

Differentiating this twice we obtain

$$\frac{dU}{dX} = \sum_0^\infty \dot{C}_{n+1} (1 - X)^n; \quad \frac{d^2U}{dX^2} = \sum_0^\infty \ddot{C}_{n+2} (1 - X)^n; \tag{7}$$

$$\dot{C}_{n+1} = -(n + 1) C_{n+1}; \quad \ddot{C}_{n+2} = (n + 1)(n + 2) C_{n+2}.$$

We note that the multiplication of the infinite power series $\sum_0^\infty A_n(1-X)^n$ and $\sum_0^\infty B_n(1-X)^n$ gives an infinite power series $\sum_0^\infty C_n(1-X)^n$, whose coefficients can conveniently be found by the Cauchy formulas

$$C_n = A_n B_0 + A_{n-1} B_1 + \ldots + A_1 B_{n-1} + A_0 B_n.$$

In accord with the notation introduced

$$\frac{dU}{dX} \cdot \frac{d^2U}{dX^2} = \sum_0^\infty [\dot{C}_{n+1} \ddot{C}_{n+2}] (1 - X)^n, \tag{8}$$

where

$$[\dot{C}_{n+1}\ddot{C}_{n+2}] = \dot{C}_1\ddot{C}_{n+2} + \dot{C}_2\ddot{C}_{n+1} + \ldots \dot{C}_n\ddot{C}_3 + \dot{C}_{n+1}\ddot{C}_2;$$

$$U\,\frac{dU}{dX} = \sum_{0}^{\infty}\,[C_n\dot{C}_{n+1}]\,(1-X)^n. \tag{9}$$

Here

$$[C_n\dot{C}_{n+1}] = C_0\dot{C}_{n+1} + C_1\dot{C}_n + \ldots + C_{n-2}\dot{C}_2 + C_{n-1}\dot{C}_1; \tag{10}$$

$$U^2 = UU = \sum_{0}^{\infty}\,[C_nC_n]\,(1-X)^n. \tag{11}$$

Similarly,

$$[C_nC_n] = C_0C_n + C_1C_{n-1} + \ldots + C_{n-1}C_1 + C_nC_0. \tag{12}$$

Substituting series (8), (9), and (11) into Eq. (5) and equating coefficients of identical powers in the parentheses $(1 - X)$, we obtain the following recurrence relation:

$$[\dot{C}_{n+1}\ddot{C}_{n+2}] + a\,[C_n\dot{C}_{n+1}] + b\,[C_nC_n] = 0. \tag{13}$$

To calculate with this equation it is necessary to have two initial conditions, e.g., C_0 and C_1, which in general must be obtained from the boundary conditions. From the second condition $U(1) = 0$, we obtain one coefficient $C_0 = 0$; the second condition is expressed in the form of the equation $\sum_{0}^{\infty} C_k = 1$, which makes C_1 potentially known also.

Thus, the problem is reduced to the solution of the following system of algebraic equations:

$$[\dot{C}_{n+1}\ddot{C}_{n+2}] + a\,[C_n\dot{C}_{n+1}] + b\,[C_nC_n] = 0 \quad (n = 0,\ 1,\ 2,\ \ldots);$$

$$\sum_{0}^{\infty} C_k = 1;\quad C_0 = 0. \tag{14}$$

With $C_0 = 0$ and a fixed n the recurrence relation (13) for $n = 0, 1, 2, \ldots$ gives each time an equation with one unknown which is expressed in terms of C_1. Limiting ourselves to a certain number of terms in series (6) we use (13) to determine the coefficients C_2, C_3, C_4, ..., C_k in terms of C_1. Substituting these into the equation $\sum_{0}^{\infty} C_k = 1$, we find C_1 and then C_2, C_3, . . ., C_k.

Specifically for $n = 0$

$$\dot{C}_1\ddot{C}_2 + aC_0\dot{C}_1 + bC_0C_0 = 0$$

or, using (7) and $C_0 = 0$, $-2C_1C_2 = 0$.

For $X = 1$ (dead end) we can write another supplementary condition $d^2U/dX^2 = 0$. Then $C_1 \neq 0$, and consequently $C_2 = \ddot{C}_2 = 0$, which enables us to express all the C_k in terms of C_1

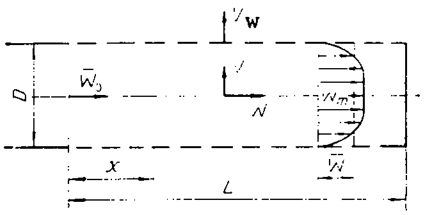

Fig. 1. Schematic diagram of porous section.

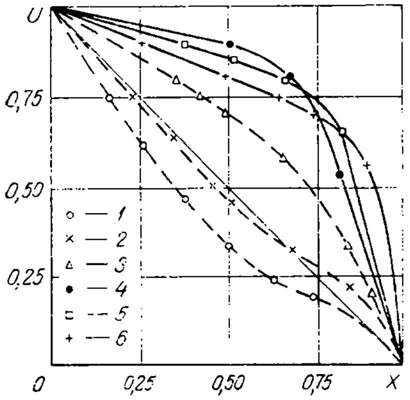

Fig. 2. Variation of relative axial velocity along a porous pipe: for $\varepsilon_f = 0.5$: 1) $L/D = 58$; 2) 43; 3) 22; for $\varepsilon_f = 0.1156$: 4) $L/D = 58$; 5) 43; 6) 22.

and the coefficients a and b which enter the equation. Limiting ourselves to a certain number of terms in series (6) and repeating the calculation described above, we find C_1 and then C_2, C_3, ..., C_k.

For n = 3,

$$U = M[120(1-X) - 20a(1-X)^3 + 10b(1-X)^4 + a^2(1-X)^5], \tag{15}$$

where $M = 1/(120 - 20a + 10b + a^2)$.

The coefficient b in (5) can be neglected if the ratio of the length of the pipe to its diameter is small. Then we obtain the linear equation

$$\frac{d^2U}{dX^2} + aU = 0.$$

The solution of this equation which satisfies the boundary conditions is

$$U = \sin\sqrt{a}(1-X)/\sin\sqrt{a}. \tag{16}$$

A similar solution was obtained by Idel'chik [7], who showed that Eq. (16) is valid only for $\sqrt{a} < \pi/2$. This same result can be obtained by the present method by limiting ourselves to n = 0 and n = 1. In this case the approximate solution of the problem is

$$U \approx [6(1-X) - a(1-X)^3]/(6-a). \tag{17}$$

This expression does not depend on the coefficient b.

System (14) can be solved by computer for practically any n. We note that the solution converges rather rapidly and within the limits of accuracy of the experiment can be restricted to the value of U for n = 3.

The variation of the dimensionless static pressure can be calculated from the equation

$$P = (U')^2(1+\zeta)/16(L/D)^2\varepsilon_f^2, \tag{18}$$

where we obtain U' by differentiating (15).

The results were tested on a hydraulic stand [8] (Fig. 1). Perforated pipes 13.8 mm in diameter with surface porosities $\varepsilon_f = 0.1156$ and $\varepsilon_f = 0.5$ were used in the experiments. The static pressure along the porous section and the pressure drop at the wall were measured in the experiment. The variation of the average axial velocity of water in the porous channel was determined from the experimental data. The length of the perforated section varied from 150 to 1000 mm. Figure 2 shows the results for pipes with various wall porosities. The curves show the dimensionless velocity averaged over the cross section as a function of the dimensionless length x/L calculated by Eq. (15). The coefficient N was determined for each case as a function of porosity, the hydraulic resistance of the wall, and the relative length of the channel. The relation U = 1 − X is valid for uniform outflow.

For the experimental data under consideration, porosities in the range $\varepsilon_f = 0.1$–0.5, and a relative channel length L/D = 15–60 we propose the relation

$$N = 12.66 + 5.6\varepsilon_f + (0.0328 - 0.0577\varepsilon_f)\,L/D.$$

The approximate method we have described for calculating the flow distribution in a porous channel has been tested over a rather wide range of controlling parameters. In all cases the experimental data and the calculated relations were in good agreement.

NOTATION

x, longitudinal coordinate, m; ρ, density of medium, kg/m^3; p, static pressure at channel wall, N/m^2; v, local radial velocity, m/sec; v_w, radial velocity at channel wall, m/sec; G, mass flow rate, kg/sec; W, local axial velocity, m/sec; $\bar{W}_0$, average velocity in entrance section of porous channel, m/sec; W_m, maximum velocity on channel axis, m/sec; $\bar{W}$, average velocity in any channel cross section, m/sec; ε_f, porosity of lateral wall; D, channel diameter, m; L, length of porous channel, m; F_k, cross-sectional area of channel, m^2; Σf_{hol}, area of holes in channel wall, m^2; $\bar{f} = \Sigma f_{hol}/F_k$; P, dimensionless pressure, $p/(^1/_2\bar{W}_0^2)\rho$; U, dimensionless average axial velocity, $U = \bar{W}/\bar{W}_0$; X, dimensionless coordinate, x/L; U'', U', derivatives wih respect to dimensionless coordinate; λ, friction factor at porous surface; λ_0, friction factor in channel with solid walls; ζ, resistance coefficient for outflow through side walls of channel; Re, Reynolds number.

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